$$\frac{\sqrt{2}}{\sqrt{3}}\log\frac{1+\sqrt{3}}{\sqrt{2}}=\frac{1}{\sqrt{2}+}\,\frac{1.2}{2\sqrt{2}+}\,\frac{3.4}{3\sqrt{2}+..}$$

$$\frac{\pi}{2}=\frac{1}{1-}\,\frac{1}{4-}\,\frac{6}{7-}\,\frac{15}{10-}\,\frac{28}{13-...}$$

$$\frac{\pi}{2\sqrt{2}}=\frac{1}{\sqrt{2}-}\,\frac{1.2}{4\sqrt{2}-}\,\frac{3.4}{7\sqrt{2}-}\,\frac{5.6}{10\sqrt{2}-..}$$

For $x=\frac{1}{2}$, $\frac{2}{\sqrt{5}}\log\frac{1+\sqrt{5}}{2}=\frac{1}{2+}\,\frac{1.2}{5+}\,\frac{3.4}{8+}\,\frac{5.6}{11+...}$

$$\frac{\pi}{3\sqrt{3}}=\frac{1}{2-}\,\frac{1.2}{7-}\,\frac{3.4}{12-}\,\frac{5.6}{17-...}$$

For $x=1$ in the expansion of $\frac{\log[x+\sqrt{(1+x^2)}]}{\sqrt{(1+x^2)}}$

$\frac{\log(1+\sqrt{2})}{\sqrt{2}}=\frac{1}{1+}\,\frac{1.2}{1+}\,\frac{3.4}{1+}\,\frac{5.6}{1+...}$, and he compares this with

$$\frac{\pi}{2}-1=\frac{1}{1+}\,\frac{1.2}{1+}\,\frac{2.3}{1+}\,\frac{3.4}{1+}\,\frac{5.6}{1+...}$$

GEOMETRY.

318. Proposed by G. W. GREENWOOD, M. A., Dunbar, Pa.

Is it possible by a straight edge and sect carrier, *i. e.*, without the use of a circle, to construct a mean proportional to two given sects?

Remark by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

The value of the length of the mean proportional can be approximately measured without the application of the circle, but it cannot be constructed by pure geometry without such application.

318. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Given three radii and the distances apart of the centers of three circles, to find the radii of the eight circles touching the three given circles.

II. Solution by G. W. GREENWOOD, Dunbar, Pa.

Consider first the problem of describing a circle touching two given circles and passing through a given point. Invert with respect to the point; the circles in general invert into circles; draw any common tangent to them

and re-invert into the original circles. The common tangent inverts into a circle through the point and tangent to the given circles. There are, in general, four solutions.

Next, let A, B, C be the centers, and a, b, c the radii, of three given circles. Suppose a is not greater than b or c. Describe circles with radii $b-a$ and $c-a$, and centers B, C, respectively. Let X be the center and x the radius of a circle through A and tangent externally to these two circles. Then a circle, center X and radius $x-a$ will be tangent to the three given circles. By obvious modifications we can, in general, get seven more circles touching the given circles.

We can get the numerical values of the radii and the positions of the centers by means of the linear relations connecting inverse figures.

It would be interesting to consider all the possible cases arising in making these constructions. For example, in the case of the escribed circles of a triangle the sides of the triangle are the limits of three pairs of circles and the nine-point circle is one of the remaining two circles.

CALCULUS.

245. Proposed by FRANCIS RUST, C. E., Allegheny, Pa.

Prove or disprove:

$$\int_0^\infty \frac{dx}{\sqrt{(x^2-1)(k^2x^2-1)}}=2F^I(k)+\sqrt{(-1)}.F^I[\sqrt{(1-k^2)}],$$

Legendre's notation, $0<k<1$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$L=\int_0^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}=\int_0^1 \frac{dx}{\sqrt{[(1-x^2)(1-k^2x^2)]}}$$

$$+\int_1^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}=F^I(k)+\int_1^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}.$$

Let $x=\dfrac{1}{ky}$. Then $\displaystyle\int_0^\infty \frac{dx}{\sqrt{[(x^2-1)(k^2x^2-1)]}}$

$$=\int_0^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}=F^I(k)+\int_1^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}.$$

$$\therefore L=2F^I(k)+\int_1^{1/k} \frac{dy}{\sqrt{[(1-y^2)(1-k^2y^2)]}}.$$

Let $y=\dfrac{1}{\sqrt{(1-k'^2z^2)}}$, where $k'=\sqrt{(1-k^2)}$.